Trigonometric Identities

In standard position, x, y axis:

$$\sin\alpha = \frac{y}{r} \qquad \cos\alpha = \frac{x}{r} \qquad \tan\alpha = \frac{y}{x} = \frac{\sin\alpha}{\cos\alpha}$$

Recall: $r^2 = x^2 + y^2$

Reciprocal Identities:

$$\sec\alpha = \frac{1}{\cos\alpha} \qquad \csc\alpha = \frac{1}{\sin\alpha} \qquad \cot\alpha = \frac{1}{\tan\alpha} = \frac{\cos\alpha}{\sin\alpha}$$

Thus: $\sin\alpha \cdot \csc\alpha = 1$ $\qquad \cos\alpha \cdot \sec\alpha = 1$ $\qquad \tan\alpha \cdot \cot\alpha = 1$

Pythagorean Identities:

$$sin^2\,\alpha + cos^2\,\alpha = 1 \qquad \sec^2\alpha = 1 + \tan^2\alpha \qquad \csc^2\alpha = 1 + \cot^2\alpha$$

Thus: $\cos^2\alpha = 1\,/\,(1 + \tan^2\alpha) = \cot^2\alpha\,/\,\csc^2\alpha = \cot^2\alpha\,/(1 + \cot^2\alpha)$

$$sin^2\,\alpha = 1/(1 + \cot^2\alpha) = \tan^2\alpha/\sec^2\alpha = \tan^2\alpha\,/(1 + \tan^2\alpha)$$

Addition and subtraction identities:

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Summary of some identities which can be concluded from the above formulas:

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha \qquad \cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$

$$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha \qquad \cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$$

$$\sin(-\alpha) = -\sin(\alpha) \qquad \cos(-\alpha) = \cos(\alpha)$$

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Also: $\sin(\alpha + \pi) = \sin\alpha \cos\pi + \cos\alpha \sin\pi = -\sin\alpha$

$$\sin(\alpha - \pi) = \sin\alpha \cos\pi - \cos\alpha \sin\pi = -\sin\alpha$$

$$\cos(\alpha + \pi) = \cos\alpha \cos\pi - \sin\alpha \sin\pi = -\cos\alpha$$

$$\cos(\alpha - \pi) = \cos\alpha \cos\pi + \sin\alpha \sin\pi = -\cos\alpha$$

The period of $\sin\alpha$ and $\cos\alpha$ is 2π, those ratios, have all a phase shift equal to half a period π.

By using the formulas above:

$$\tan(\alpha + \pi) = \tan\alpha \qquad \tan(\pi - \alpha) = \tan\alpha$$

$$\cot(\alpha + \pi) = \cot\alpha \qquad \cot(\pi - \alpha) = \cot\alpha$$

Remember that, the period of $\tan\alpha$ and $\cot\alpha$, is just π.

Double angle Identities:

$$\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

Note 1: $\cos^2\alpha = (\cos 2\alpha + 1)/2$ and, $\sin^2\alpha = (1 - \cos 2\alpha)/2$

Circle trigonometry:

If a circle is divided into "n" equal central sectors, all of its quantities will be divided equally by n:

$$\frac{1}{n} = \frac{\theta°}{360°} = \frac{\theta^{Rad}}{2\pi} = \frac{l}{2\pi r} = \frac{A}{\pi r^2}$$

Recall: a circle is 360 Degrees or 2π Radians, the circumference is $2\pi r$, and the area πr^2. "r" is the radius.

A sector with a central angle $\theta°$ or θ in Radians, has an arc length l, and the area A:

According the formulas above, we can briefly conclude:

$$\frac{\theta°}{180°} = \frac{\theta^{Rad}}{\pi} \qquad l = r\theta^{Rad} \qquad A = 1/2\theta^{Rad}r^2$$

Note: The area of the central triangle (the triangle inscribed in the sector is:

$$A = 1/2r^2 sin\theta$$

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